MA135—VECTORS AND MATRICES
Example Sheet 1

All questions in sections A and B must be handed in to your supervisor via the supervisor's pigeon loft by **3pm Monday, week 2.** Section C questions should be **attempted** by students who hope to get a 1st or 2:1 degree, but are not to be handed in.

Section A

A1. Let $\alpha = (1 + 2i)$, $\beta = (3 - 4i)$ and $\gamma = (7 - i)$. Evaluate the following:

(i) $\alpha + \gamma$ (ii) $\alpha\beta$ (iii) $1/\alpha$

(iv) $\operatorname{Im}(\beta/\gamma)$ (v) $\operatorname{Re}(\alpha^2 - \beta)$ (vi) $\overline{\alpha}\beta$.

A2. Which of the following are true/false (with reasons):

(i) Every real number is a complex number.
(ii) The square of an imaginary number is a negative real number.
(iii) There is a complex number which is both real and imaginary.

A3. For which positive integral values of n is i^n real?

Section B

B1. Solve the equation $(5 - i)X + (2 + i)(-2 + 3i) = \overline{i - 1}$.

B2. Write

$$\frac{1}{\cos\theta + i\,\sin\theta}$$

in the form $a + i\,b$. (do this question using the definition of reciprocal).

B3. Which of the following are true **for all** complex numbers α and β?

(a) $|\alpha + \beta| = |\alpha| + |\beta|$.
(b) $\operatorname{Re}(\alpha - \beta) = \operatorname{Re}(\alpha) - \operatorname{Re}(\beta)$.
(c) $\operatorname{Im}(\alpha\beta) = \operatorname{Im}(\alpha)\operatorname{Im}(\beta)$.

(If you say 'true', then you should show why it is true. If you say 'false', you have to give a counter-example).

Section C

You will need the Fundamental Theorem of Algebra to answer part (ii) of the following question. The Fundamental Theorem of Algebra, which we will shorly cover in the lectures, says the following: if f is a polynomial of degree n with complex coefficients, then f has n complex roots (counting multiplicities).

C1. Let $f = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_0$ be a polynomial where the coefficients $a_0, \dots, a_n$ are real and $a_n \neq 0$.

(i) Show that if α is a complex root of f then its conjugate $\overline{\alpha}$ is also a root of f.
(ii) Suppose that the degree n is odd. Show that f must have at least one real root.